LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – APRIL 2010

ST 2902 - PROBABILITY THEORY AND STOCHASTIC PROCESSES

Date & Time: 26/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART-A

Answer all the questions **10 x 2 = 20**

1) If $A \subset B$, show that $P(A) \leq P(B)$.
2) If A and B are independent events, show that A^C and B^C are also independent .
3) Define the MGF of a random variable X .
4) Find the constant C ,if the following represents the probability distribution of a random variable $p(x) = C\left(\frac{1}{3}\right)^x$, x= 1,2,3….
5) Define a Markov chain.
6) When do you say that state i of a Markov chain is transient or recurrent ?
7) The joint pdf of the random variables X_1 and X_2 is given by $f(x_1,x_2) = 6x_2$ $0< x_2 < x_1 < 1$. Find the marginal pdf of X_1 .
8) Define the covariance between two variables X_1 and X_2. What happens to the covariance when they are independent ?
9) State any two properties of normal distribution .
10) Explain Renewal process .

PART-B

ANSWER ANY 5 QUESTIONS **5 X 8 = 40**

11) A bowl contains 16 chips of which 6 are red , 7 are white and 3 are blue . If 4 chips are taken at random and without replacement find the probability that
    a) all are red
    b) none of them is red
    c) atleast 1 chip of each colour .
    d) exactly 2 of them are blue .
12) Show that the distribution function is non decreasing and right continuous .
13) Let the joint pdf of the two random variables X_1 and X_2 be $f(x_1, x_2) = x_1+x_2$, $0< x_1< 1$, $0<x_2< 1$. obtain $E[X_2 / X_1=x_1]$ and $var[X_2 / X_1=x_1]$.
14) State and prove Chapman Kolmogrov equation for a discrete time Markov chain .
15) Obtain the MGF of binomial distribution . Hence obtain mean and variance.
16) Derive the Kolmogrov Backward diffrential equations for a Birth death process.
17) State and prove Bayes theorem .
18) If the states i and j communicate , then show that d (i) = d (j) .

PART-C

ANSWER ANY TWO QUESTIONS **2 X 20 = 40**

19) a) State and prove addition theorem for n events .

b) Let $\{A_n\}$ be an increasing sequence of events . show that $P(\lim A_n) = \lim P(A_n)$. Deduce the result for decreasing events . (10+10)

20) a) State and prove central limit theorem for a sequence of i.i.d random variables .

b) Let X_1, X_2 be independent random variables with pdf 's

$$f_1(x_1) = \frac{e^{-x_1} {x_1}^{m-1}}{(m-1)!} \qquad 0 < x_1 < \infty$$

$$f_2(x_2) = \frac{e^{-x_2} {x_2}^{n-1}}{(n-1)!} \qquad 0 < x_2 < \infty$$

Show that $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_1+X_2}$ are independent . (10+10)

21) a)State the postulates of poisson process and hence obtain an expression for $p_n(t)$.

b) Obtain the stationary distribution for a Markov chain with transition probability matrix p and states 0,1,2 ,3….. (12+8)

$$p = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

22)a) Show that under certain conditions (to be stated) binomial distribution tends to poisson distribution .

b) Show that for normal distribution $\mu_{2n} = 1.3.5.........(2n-1)\sigma^{2n}$. n = 1,2,---

c) Let the probability mass function p(x) be positive on 1,2,3........ Given that

$p(x) = \frac{4}{x}\, p(x-1)$, x = 1,2,3 ....Find $p(x)$. (7+7+6)

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